

# Ricci Flat Rotating Black Branes with a Conformally Invariant Maxwell Source

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We consider Einstein gravity coupled to an  $U(1)$  gauge field for which the density is given by a power of the Maxwell Lagrangian. In  $d$ -dimensions the action of Maxwell field is shown to enjoy the conformal invariance if the power is chosen as  $d/4$ . We present a class of charge rotating solutions in Einstein-conformally invariant Maxwell gravity in the presence of a cosmological constant. These solutions may be interpreted as black brane solutions with inner and outer event horizons or an extreme black brane depending on the value of the mass parameter. Since we are considering power of the Maxwell density, the black brane solutions exist only for dimensions which are multiples of four. We compute conserved and thermodynamics quantities of the black brane solutions and show that the expression of the electric field does not depend on the dimension. Also, we obtain a Smarr-type formula and show that these conserved and thermodynamic quantities of black branes satisfy the first law of thermodynamics. Finally, we study the phase behavior of the rotating black branes and show that there is no Hawking–Page phase transition in spite of conformally invariant Maxwell field.

## I. INTRODUCTION

The conjectured equivalence of string theory on anti-de Sitter (AdS) spaces (times some compact manifold) and certain superconformal gauge theories living on the boundary of AdS [1] has lead to an increasing interest in asymptotically anti-de Sitter black holes. This conjecture is now a fundamental concept that furnishes a means for calculating the action and conserved quantities intrinsically without reliance on any reference spacetime [2]. Also, coupling between gravity and gauge fields is a common feature of many unification theories in higher dimensions [3]. In supergravity theories, such gauge fields are frequently essential in order to complete the multiple structure and to guarantee the invariance of the Lagrangian with respect to local supersymmetry transformations [4]. They can provide, in some cases, a dynamical mechanism for the compactification of

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extra dimensions [5].

In this supersymmetric context, they can lead to a geometric interpretation in the superspace. The simplest example of coupling of a gauge field to gravity is the Einstein-Maxwell system in four dimensions, whose extension to higher dimensions can lead to some interesting features: its reduction to four dimensions implies a non-trivial coupling of the electromagnetic field to gravity. In the conventional, straightforward generalization of the Maxwell field to higher dimensions one essential property of the electromagnetic field is lost, namely, conformal invariance.

The Reissner-Nordström black hole, the first solution for which the matter source is conformally invariant, is an electrically charged but non-rotating black hole solution in four dimensions. There exists a conformally invariant extension of the Maxwell (CIM) action in higher dimensions, if one uses the lagrangian of the  $U(1)$  gauge field in the form:

$$I_{CIM} = \alpha \int d^d x \sqrt{-g} (F_{\mu\nu} F^{\mu\nu})^{d/4} \quad (1)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the Maxwell tensor and  $\alpha$  is a constant. It is straightforward to show that the action (1) is invariant under conformal transformation ( $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$  and  $A_\mu \rightarrow A_\mu$ ) and for  $d = 4$ , the action (1) reduces to the Maxwell action as it should be. The energy-momentum tensor associated to  $I_{CIM}$  is given by

$$T_{\mu\nu} = \alpha \left( d F_{\mu\rho} F_\nu{}^\rho F^{(d/4)-1} - g_{\mu\nu} F^{d/4} \right) \quad (2)$$

where  $F = F_{\mu\nu} F^{\mu\nu}$  and it is easy to show that  $T_\mu^\mu = 0$  [6]. As one can see the clue of the conformal invariance lies in the fact that we have considered power of the Maxwell invariant. This idea has been applied in the case of scalar field for which it has been shown that particular power of the massless Klein-Gordon Lagrangian exhibits conformal invariance in arbitrary dimension [7]. It would be interesting to see whether black hole solutions can also be obtained in this case.

In what follows, we have been considered the action (1) as the matter source coupled to the Einstein gravity to take advantage of the conformal symmetry to construct the analogues of the four-dimensional Reissner-Nordström black hole solutions in higher dimensions. The form of the energy-momentum tensor (2), automatically restricts the dimensions to be only multiples of four.

However, the black hole solutions which have been studied so far represent only a special case of the general black hole solution since rotation has been ignored [5, 8]. To see whether qualitative properties of generic black holes depend on the matter content, one must consider rotating charged black holes [9–11]. This is the subject of the present investigation. Indeed, in this paper we obtain a class of higher dimensional solution of rotating black branes with a conformally

invariant Maxwell source in the presence of cosmological constant. These solutions contain three conserved quantities, the mass  $M$ , electrical charge  $Q$ , and angular momentum  $J$ . By calculation of other thermodynamic quantities, we show that thermodynamic and conserved quantities satisfy the first law of thermodynamics. Also, we investigate the stability of charged rotating black brane solutions in the canonical and grand canonical ensembles. In fact, the Hawking-Page phase transition for the Schwarzschild-AdS black hole [12] does not occur for asymptotically locally anti-de Sitter black holes whose horizons have vanishing or negative constant curvature, and they are thermodynamically –locally– stable since they have a positive heat capacity [13]. The inclusion of electric charge further modifies the thermodynamical properties of black holes and a more complex phase structure arises, allowing for an analogous of the Hawking-Page phase transition [14].

The rest of the paper is organized as follows. In Sec. II, we give a brief review of the field equations of Einstein gravity with conformally invariant Maxwell field in the presence of cosmological constant and review the counterterm method for calculating the conserved quantities of the solutions. In Sec. III, we obtain the  $(4p + 4)$ -dimensional static solutions of field equations, which are asymptotically (anti)-de sitter or flat. Then, we present a new class of charged rotating black brane solutions and investigate their properties in Sec. IV. In Sec. V we obtain mass, angular momentum, entropy, temperature, charge, and electric potential of the rotating black brane solutions and show that these quantities satisfy the first law of thermodynamics. We also perform a local stability analysis of the black branes in the canonical and grand-canonical ensembles. We finish our paper with some concluding remarks.

## II. FIELD EQUATIONS IN EINSTEIN-CONFORMALLY INVARIANT MAXWELL GRAVITY

In  $d = 4$ , the action of Einstein-Maxwell is trivial, but for  $d = 4p + 4$  dimensions with  $p \in \mathbf{N}$ , we consider the Einstein action with a conformally invariant Maxwell action given by

$$I_G = - \frac{1}{16\pi} \int_{\mathcal{M}} d^{4p+4}x \sqrt{-g} [\mathcal{R} - 2\Lambda - \alpha(F^{\mu\nu}F_{\mu\nu})^{p+1}] - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^{4p+3}x \sqrt{-\gamma} \Theta(\gamma), \quad (3)$$

where  $\Lambda$  is the negative cosmological constant which is equal to  $-(4p+3)(2p+1)/l^2$  for asymptotically AdS solutions, where  $l$  is a scale length factor;  $\alpha$  is a constants and  $\mathcal{R}$  is scalar curvature. The second integral in Eq. (4) is a boundary term which is chosen such that the variational principle is well defined [15]. In this term  $\gamma_{ij}$  is an induced metric on the boundary  $\partial\mathcal{M}$  and  $\Theta$  is the trace of

the extrinsic curvature  $\Theta_{\mu\nu}$  of any boundary of the manifold  $\mathcal{M}$ . Varying the action with respect to the gauge field  $A_\mu$  and the metric  $g_{\mu\nu}$ , the field equations are obtained as

$$\partial_\mu (\sqrt{-g} F^{\mu\nu} F^\rho) = 0, \quad (4)$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 2\alpha \left[ (p+1) F_{\mu\rho} F_\nu{}^\rho F^p - \frac{1}{4} g_{\mu\nu} F^{p+1} \right], \quad (5)$$

respectively.

In general the action  $I_G$ , is divergent when evaluated on the solutions, as is the Hamiltonian and other associated conserved quantities. A systematic method of dealing with this divergence in Einstein gravity is through the use of the counterterms method inspired by the anti-de Sitter conformal field theory (AdS/CFT) correspondence [1]. In Einstein gravity, there exists only one counterterm and therefore the total finite action is

$$I = I_G + I_{\text{ct}}, \quad (6)$$

where the counterterm action is

$$I_{\text{ct}} = -\frac{2p+1}{4\pi l} \int_{\partial\mathcal{M}} d^{4p+3}x \sqrt{-\gamma}. \quad (7)$$

Having the total finite action one can construct a divergence-free stress energy tensor by use the Brown-York definition [16]. One can show that the finite stress energy tensor is

$$T^{ab} = \frac{1}{8\pi} \left[ \Theta^{ab} - \Theta \gamma^{ab} - \frac{2(2p+1)}{l} \gamma^{ab} \right]. \quad (8)$$

To compute the conserved charges of the spacetime, we choose a spacelike hypersurface  $\mathcal{B}$  in  $\partial\mathcal{M}$  with metric  $\sigma_{ij}$ , and write the boundary metric in *ADM* (Arnowitt-Deser-Misner) form

$$\gamma_{ab} dx^a dx^a = -N^2 dt^2 + \sigma_{ij} (d\varphi^i + V^i dt) (d\varphi^j + V^j dt), \quad (9)$$

where the coordinates  $\varphi^i$  are the angular variables parameterizing the hypersurface of constant  $r$  around the origin, and  $N$  and  $V^i$  are the lapse and shift functions respectively. When there is a Killing vector field  $\xi$  on the boundary, then the quasilocal conserved quantities associated with the stress energy tensors of Eq. (8) can be written as

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{4p+2}\varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (10)$$

where  $\sigma$  is the determinant of the metric  $\sigma_{ij}$ , and  $n^a$  is the timelike unit normal vector to the boundary  $\mathcal{B}$ . Hereafter we set  $\alpha = (-1)^p$  without loss of generality and consequently the energy density (the  $T_{\hat{0}\hat{0}}$  component of the energy-momentum tensor in the orthonormal frame) is positive.

### III. THE STATIC SOLUTIONS

Here we want to obtain the  $(4p+4)$ -dimensional static solutions of Eq. (4) and (5), which are asymptotically (anti)-de sitter or flat. We assume that the metric has the following form:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{4p+2}^2 \quad (11)$$

where  $d\Omega_{4p+2}^2$  is

$$d\Omega_{4p+2}^2 = \begin{cases} d\theta_1^2 + \sum_{i=2}^{4p+2} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2 & k = 1 \\ d\theta_1^2 + \sinh^2 \theta_1 d\theta_2^2 + \sinh^2 \theta_1 \sum_{i=3}^{4p+2} \prod_{j=2}^{i-1} \sin^2 \theta_j d\theta_i^2 & k = -1 \\ \sum_{i=1}^{4p+2} d\zeta_i^2 & k = 0 \end{cases}$$

which represents the line element of an  $(4p+2)$ -dimensional hypersurface with constant curvature  $(4p+2)(4p+1)k$  and volume  $V_{4p+2}$ . We use the gauge potential ansatz

$$A_\mu = h(r)\delta_\mu^0 \quad (12)$$

in conformally invariant Maxwell equation (4). we obtain

$$h(r) = \frac{-q}{r},$$

where  $q$  is an integration constant which is related to the electric charge parameter. Also, the conformally invariant Maxwell equation implies that the electric field in  $(4p+4)$ -dimensions is given by

$$F_{tr} = \frac{q}{r^2}. \quad (13)$$

It is interesting to note that the expression of the electric field does not depend on the dimension and its value coincides with the Reissner-Nordström solution in four dimensions.

To find the metric function  $f(r)$ , one may use any components of Eq. (5). The simplest equation is the  $rr$  component of these equations, which can be written as

$$f'(r) + \frac{4p+1}{r} [f(r) + k] + \frac{2^p q^{2p+2}}{r^{4p+3}} + \frac{\Lambda r}{2p+1} = 0, \quad (14)$$

where the prime denotes a derivative with respect to  $r$ . The solutions of Eq. (14) can be written as

$$f(r) = k + \frac{r^2}{l^2} - \frac{m}{r^{4p+1}} + \frac{2^p q^{2p+2}}{r^{4p+2}}, \quad (15)$$

where  $m$  is other integration constant proportional to the mass parameter. One can check that the solution given by Eq. (15) satisfies all the components of the field equations (5). The metric function  $f(r)$ , presented here, differ from the standard higher-dimensional solutions; we find that the electric charge term in the metric function goes as  $r^{-(4p+2)}$  and in the standard case is  $r^{-2(4p+1)}$ . Hereafter we set  $k = 0$  and investigate the properties of the boundary flat black brane solutions.

#### IV. THE $(4p+4)$ -DIMENSIONAL CHARGED ROTATING BLACK BRANES

The metric of  $(4p+4)$ -dimensional asymptotically AdS charged rotating black brane with  $\kappa$  rotation parameters and flat boundary at constant  $t$  and  $r$  may be written as [17]

$$ds^2 = -f(r) \left( \Xi dt - \sum_{i=1}^{\kappa} a_i d\phi_i \right)^2 + \frac{r^2}{l^4} \sum_{i=1}^{\kappa} (a_i dt - \Xi l^2 d\phi_i)^2 + \frac{dr^2}{f(r)} - \frac{r^2}{l^2} \sum_{i < j}^{\kappa} (a_i d\phi_j - a_j d\phi_i)^2 + r^2 dX^2, \quad (16)$$

where  $f(r)$  is the same as  $f(r)$  given in Eq. (15) with  $k = 0$ ,  $\Xi = \sqrt{1 + \sum_i^k a_i^2/l^2}$  and  $dX^2$  is the Euclidean metric on the  $(4p+2-\kappa)$ -dimensional submanifold. The rotation group in  $d$ -dimensions is  $SO(4p+3)$  and therefore the maximum number of rotation parameters in  $(4p+4)$ -dimensions is  $[\frac{4p+3}{2}]$ , where  $[z]$  denotes the integer part of  $z$ .

The gauge potential is

$$A_\mu = \frac{-q}{r} (\Xi \delta_\mu^0 - \delta_\mu^i a_i) \quad (\text{no sum on } i). \quad (17)$$

##### A. Properties of the solutions

In order to study the general structure of these solutions, we first look for the essential singularities. After some algebraic manipulation, one can show that the Kretschmann scalar in  $(4p+4)$ -dimensions,

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{8(p+1)(4p+3)}{l^4} + \frac{4(4p+3)(4p+1)(2p+1)^2 m^2}{r^{8p+6}} - \frac{2^{p+4}(p+1)(4p+3)(2p+1)(4p+1)mq^{2p+2}}{r^{8p+7}} + \frac{2^{2p+3}(p+1)(2p+1)(16p^2+24p+7)q^{4p+4}}{r^{8p+8}}, \quad (18)$$

diverges at  $r = 0$  and is finite for  $r \neq 0$ . Thus, there is an curvature timelike singularity located at  $r = 0$ . In the case of a vanishing cosmological constant, the solutions given in Eqs. (15)

and (16) are not well-behaved asymptotically and for the case of a positive cosmological constant  $[\Lambda = (4p+3)(2p+1)/l^2]$  the solutions are asymptotically dS, but the singularity is naked (see the Appendix of [10]).

It is proved that a stationary black hole event horizon should be a Killing horizon in the four-dimensional Einstein gravity [18]. In our solutions the Killing vector,

$$\chi = \partial_t + \sum_i^k \Omega_i \partial_{\phi_i}, \quad (19)$$

is the null generator of the event horizon. The angular velocities  $\Omega_i$  are [19]

$$\Omega_i = \frac{a_i}{\Xi l^2}. \quad (20)$$

The temperature may be obtained through the use of the definition of surface gravity,

$$T_+ = \frac{1}{\beta_+} = \frac{1}{2\pi} \sqrt{-\frac{1}{2} (\nabla_\mu \chi_\nu) (\nabla^\mu \chi^\nu)}$$

where  $\chi$  is the Killing vector (19). One obtains

$$T_+ = \frac{(4p+3)r_+^{4p+4} - 2^p q^{2p+2} l^2}{4\pi l^2 \Xi r_+^{4p+3}}. \quad (21)$$

The event horizon of solutions are located at the root(s) of  $g^{rr} = f(r) = 0$ . Indeed, the metric of Eqs. (15) and (16) has two inner and outer event horizons located at  $r_-$  and  $r_+$ , provided the mass parameter  $m$  is greater than  $m_{\text{ext}}$  given as

$$m_{\text{ext}} = \frac{4(p+1)}{l^2} \left( \frac{2^p l^2 q_{\text{ext}}^{2p+2}}{4p+3} \right)^{(4p+3)/(4p+4)}. \quad (22)$$

The solutions present naked singularity for  $m < m_{\text{ext}}$  and when  $m = m_{\text{ext}}$ , we have an extreme black brane with horizon radius

$$r_{+\text{ext}} = \left( \frac{2^p l^2 q_{\text{ext}}^{2p+2}}{4p+3} \right)^{1/(4p+4)}. \quad (23)$$

## V. THERMODYNAMICS

### A. Conserved and thermodynamic quantities

Usually entropy of the black holes satisfies the so-called area law of entropy which states that the black hole entropy equals to one-quarter of horizon area [20]. Since the area law of the entropy

is universal, and applies to all kinds of black holes/branes in Einstein gravity [20], therefore the entropy per unit volume  $V_{4p+2}$  is equal to one-quarter of the area of the horizon, i.e.,

$$S = \frac{\Xi}{4} r_+^{2(2p+1)}. \quad (24)$$

where  $V_{4p+2}$  is the volume of the boundary at constant  $t$  and  $r$ .

The electric charge per unit volume  $V_{4p+2}$  of the black brane,  $Q$ , can be found by calculating the flux of the electromagnetic field at infinity, yielding

$$Q = \frac{2^p(p+1)\Xi q^{2p+1}}{4\pi}. \quad (25)$$

Also, the electric potential  $U$ , measured at infinity with respect to the horizon, is defined by [21]

$$U = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r=r_+}, \quad (26)$$

where  $\chi$  is the null generator of the horizon given by Eq. (19). One finds

$$U = \frac{q}{\Xi r_+}, \quad (27)$$

which is independent of dimensions.

The present spacetime (16), have boundaries with timelike ( $\xi = \partial/\partial t$ ) and rotational ( $\varsigma = \partial/\partial \varphi$ ) Killing vector fields. From Eq. (10), one obtains the quasilocal mass and angular momentum

$$M = \int_{\mathcal{B}} d^{4p+2} \varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (28)$$

$$J = \int_{\mathcal{B}} d^{4p+2} \varphi \sqrt{\sigma} T_{ab} n^a \varsigma^b, \quad (29)$$

provided the hypersurface  $\mathcal{B}$  contains the orbits of  $\varsigma$ . Using the Eqs. (28) and (29), we find the total mass per unit volume  $V_{4p+2}$  of the solutions to be given by

$$M = \frac{[(4p+3)\Xi^2 - 1]}{16\pi} m, \quad (30)$$

while the angular momenta per unit volume  $V_{4p+2}$  are given by

$$J_i = \frac{(4p+3)\Xi m}{16\pi} a_i. \quad (31)$$

## B. Energy as a function of entropy, angular momenta and charge

Now, we check the first law of thermodynamics for our solutions in Einstein-conformally invariant Maxwell gravity. In order to do this, we obtain the mass  $M$  as a function of the extensive



quantities  $S$ ,  $\mathbf{J}$  and  $Q$ . Using the expression for the entropy, the mass, the angular momenta, and the charge given in Eqs. (30), (31), (24), (25), and the fact that  $f(r_+) = 0$ , one can obtain a Smarr-type formula as

$$M(S, J, Q) = \frac{[(4p+3)Z-1]J}{(4p+3)l\sqrt{Z(Z-1)}}, \quad (32)$$

where  $J = |\mathbf{J}| = \sqrt{\sum_i^k J_i^2}$  and  $Z = \Xi^2$  is the positive real root of the following equation

$$S^{(4p+3)/(4p+2)} + \frac{\pi Q l^2}{p+1} \left( \frac{\pi^2 Q^2}{2^{2p}(p+1)^2 S} \right)^{1/(4p+2)} - \frac{2^{2p/(2p+1)} \pi l J Z^{1/(8p+4)}}{(4p+3)\sqrt{Z-1}} = 0. \quad (33)$$

One may then regard the parameters  $S$ ,  $\mathbf{J}$  and  $Q$  as a complete set of extensive parameters for the mass  $M(S, \mathbf{J}, Q)$  and define the intensive parameters conjugate to  $S$ ,  $\mathbf{J}$  and  $Q$ . These quantities are the temperature, the angular velocities and the electric potential. It is a matter of straightforward calculations to obtain

$$\Omega_i = \left( \frac{\partial M}{\partial J_i} \right)_{S, Q}, \quad T = \left( \frac{\partial M}{\partial S} \right)_{J, Q}, \quad U = \left( \frac{\partial M}{\partial Q} \right)_{S, J}. \quad (34)$$

Using equations (24), (25) and (31), it is easy to show that the intensive quantities calculated from equation (34) coincide with equations (20), (21) and (27). Thus, these thermodynamics quantities satisfy the first law of thermodynamics

$$dM = T dS + \sum_{i=1}^k \Omega_i dJ_i + U dQ. \quad (35)$$

### C. Stability in the canonical and the grand-canonical ensemble

Finally, we investigate the stability of charged rotating black brane solutions of Einstein-conformally invariant Maxwell gravity. The local stability can in principle be carried out by finding the determinant of the Hessian matrix of  $M(S, Q, \mathbf{J})$  with respect to its extensive variables  $X_i$ ,  $\mathbf{H}_{X_i X_j}^M = [\partial^2 M / \partial X_i \partial X_j]$  [21]. In our case the mass is a function of the entropy, the angular momenta and the charge. The number of thermodynamic variables depends on the ensemble that is used. In the canonical ensemble, the charge and the angular momenta are fixed parameters, and therefore the positivity of the heat capacity  $C_{\mathbf{J}, Q} = T_+ / (\partial^2 M / \partial S^2)_{\mathbf{J}, Q}$  is sufficient to ensure the local stability.  $(\partial^2 M / \partial S^2)_{\mathbf{J}, Q}$  at constant charge and angular momenta is

$$\left( \frac{\partial^2 M}{\partial S^2} \right)_{\mathbf{J}, Q} = \frac{(4p+3) \left( r_+^{4(p+1)} + 2^p q^{2(p+1)} l^2 \right)^{-1}}{\pi \Xi^2 l^2 [(p+1)\Xi^2 + 1] r_+^{8p+5}} \Upsilon. \quad (36)$$

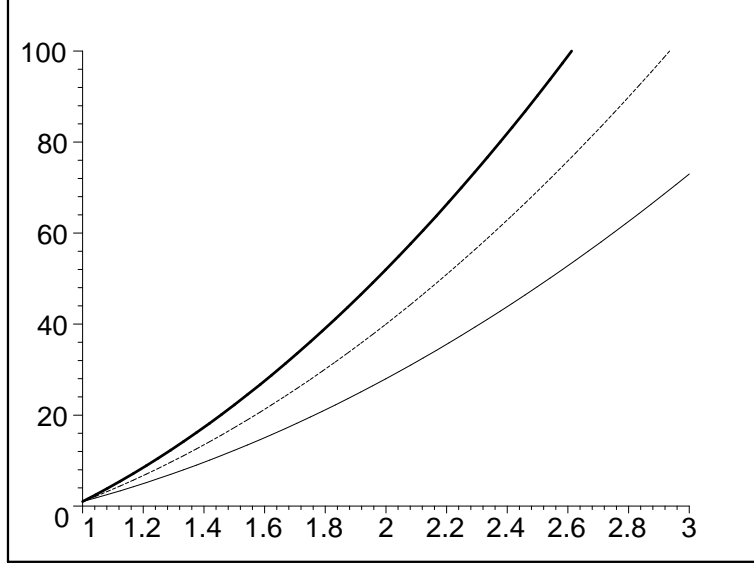


FIG. 1:  $\Upsilon$  versus  $\Xi$  for  $r_+ = 1$ ,  $l = 0.1$ ,  $q = 0.1$  and  $p = 1$  (continuous line),  $p = 2$  (dotted line) and  $p = 3$  (bold line).

where

$$\begin{aligned} \Upsilon = & 2^{2p} l^4 \Xi^2 q^{4(p+1)} + \frac{2^{p+1}}{(2p+1)} q^{2(p+1)} l^2 r_+^{4(p+1)} [2(p+1) - \Xi^2] + \\ & [4(p+1)(\Xi^2 - 1) + \Xi^2] r_+^{8(p+1)} \end{aligned} \quad (37)$$

The heat capacity is positive for  $m \geq m_{\text{ext}}$ , where the temperature is positive. This fact can be seen easily for  $\Xi = 1$ , where the  $\Upsilon$  is positive and then  $(\partial^2 M / \partial S^2)_{\mathbf{J}, Q}$  is positive too. Also, one may see from Fig. 1 that the  $\Upsilon$  increases as  $\Xi$  increases, and therefore it is always positive. Thus, the black brane is stable in the canonical ensemble. In the grand-canonical ensemble, after some algebraic manipulation, we obtain

$$\mathbf{H}_{S, \mathbf{J}, Q}^M = \frac{64\pi \left[ (4p+3) r_+^{4(p+1)} + (2p+1) 2^p l^2 q^{2(p+1)} \right] \left[ r_+^{4(p+1)} + 2^p l^2 q^{2(p+1)} \right]^{-1}}{[(4p+1)\Xi^2 + 1] \left[ (4p+3)(2p+1)(p+1) 2^p l^2 \Xi^6 r_+^{8p+5} q^{2p} \right]}. \quad (38)$$

It is easy to see  $\mathbf{H}_{S, \mathbf{J}, Q}^M$  is positive for all the allowed values of  $q \leq q_{\text{ext}}$ . Thus, the  $(4p+4)$ -dimensional asymptotically AdS charged rotating black brane is locally stable in the grand-canonical ensemble.

## VI. CLOSING REMARKS

First of all, we considered Einstein gravity coupled to a conformally invariant Maxwell field. In  $d$ -dimensions the action of Maxwell field is shown to enjoy the conformal invariance if the power

is chosen as  $d/4$ . Then, For obtaining finite action, we combined well-defined Einstein-conformally invariant Maxwell action with conterterm action for which, the last action arise from AdS/CFT correspondence. After that, we found a class of charge rotating solutions in Einstein-conformally invariant Maxwell gravity in the presence of a cosmological constant. These solutions may be interpreted as black brane solutions with inner and outer event horizons or an extreme black brane when  $m > m_{\text{ext}}$  or  $m = m_{\text{ext}}$ , respectively. The black holes presented here differ from the standard higher-dimensional solutions [11] since (a) the scalar curvature of the spacetimes is only related to cosmological constant, and (b) the electric charge term in the metric coefficient goes as  $r^{-(4p+2)}$  and in the standard case is  $r^{-2(4p+1)}$ .

Also, we computed physical quantities of the black brane solutions such as the temperature, the angular velocity, the electric charge and the potential, and showed that the expression of the potential does not depend on the dimension. Also, we obtained a Smarr-type formula and found that for these black brane solutions, these conserved and thermodynamic quantities satisfy the first law of thermodynamics. Finally, we studied the phase behavior of the rotating black branes and showed that there is no Hawking–Page phase transition in spite of conformally invariant Maxwell field. Indeed, we investigated stability of black brane in both the canonical and grand-canonical ensembles and show that the system is stable in the whole phase space.

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